Strategies of Knowing and Understanding Mathematics

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Abstract

Mathematics is a subject which is imperative in every aspect of life worldwide. It is one of the main subjects that students struggle with in schools. This research was done after observing students from a wide variety of schools as well as the author's own experience and past struggles with the subject. It precisely points out specific problems that pose a challenge on students and strategies of how to enable students to break free and achieve success in mathematics. The research is useful for a wide range of ages.

Through the years mathematics has be taught using many different textbooks most of which start with a uniform order of topics. This has done little to help students who lag behind. However, this research has an effective and unique order of only the main topics involved in the strategy. The main objective is to enable students to not only understand the subject when it is taught in their respective learning institutions, but also to enable them to implement the strategies in their self-study. This is critical as most students do not have enough time and resources to afford a professional private instructor.

I. INTRODUCTION

This research on how to know mathematics started in 2008 from September to November. In the years leading to that, during the years in school from grade 8 to 9, as well as part of grade 10, I had problems solving mathematics. Fortunately, towards the end of grade 10 I realized the simple rules that are in mathematics. At that time I had known how to solve mathematics and developed an interest to help others know the simples rules so that together we can be successful in mathematics and even other academic areas.

The methods in this research were tested on several students who struggled to understand mathematical concepts. After implementing the methods in this research article, their performance in mathematics improved. I invite you to enjoy studying this research article on how to know and understand mathematics.

II. METHODS

Lecture the following subjects: Operations, BODMAS, Number-line, Variables, Equations and Inequalities to students and give exercises to practice and evaluate their outcomes.

Illustrating how to effectively apply the guidelines in the research and following the examples given.

III. PROCEDURE

1. .BODMAS

The word BODMAS means Brackets of Division, Multiplication, Addition, and Subtraction. The first letter f each word makes up the word BODMAS.

B: Brackets

D: Division

A: Addition

O: Of

M: Multiplication

S: Subtraction

BODMAS is a formula; it is used to calculate mathematics which has many operations. In short, BODMAS is used to calculate long mathematics.

Example: 4 + 2 × 6 ÷ 3 – 4 =

For you to find the answer for this expression, you start with calculating the numbers that have an operation in between which comes first in the word BODMAS.

In the example 4 + 2 × 6 ÷ 3 – 4 = , you should not calculate it just the way it is, instead, use BODMAS to find the right answer.

Since the example does not have brackets, you start with D: dividing, and then multiplying, after that, you add, and at last you subtract. You have to follow the order of the word BODMAS.

Divide first

4 + 2 × 6 ÷ 3 – 4 =

Then multiply

= 4 + 2 × 2 – 4

Then add

= 4 + 4 – 4

At last, subtract

= 8 – 4

= 4

For a question or expression which has brackets, you have to start with what is in the brackets first.

Example:

(2 + 3) – 4 (2 ÷ 2)

= 5 – 4 (1)

Brackets also mean x or multiplication. For example (1) is the same as writing 4 × 1 = 1

When calculating long questions, you should always remember the word BODMAS. You should not swap or forget any letter. If you forget one letter or, if you put the letter somewhere where it is not supposed to be, you will be find wrong answers. So, do not swap the letters, instead, master the word BODMAS exactly the way it is.

2. NUMBERLINE

In all calculations that have addition (+) and those that have subtraction (-) a number-line is used. Always use a number-line when adding or subtracting.

*In the middle of a number-line there is a zero

*On the right side of the number-line there are numbers with a positive/plus (+) sign. However, there are certain number-lines which have numbers on the right side without any sign written; those numbers are positive numbers as long as they are on the positive side of the number-line.

*On the left side of the number-line there are numbers with a negative (-) sign.

*The value of numbers on the number-line increase as you move to the right side. For example

-5 -4 -3 -2 -1 0 + 1
+2 +3 +4 +5

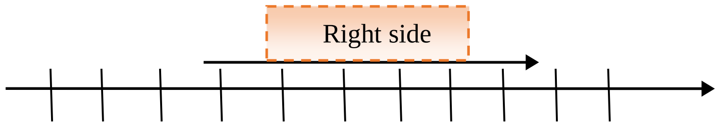
*So, +1 is greater than -3 because +1 is on the right side of -3 on the number-line.

* +4 is greater than -5 because +4 it is on the right of -5.

*All positive numbers (+) or numbers without any sign are greater than all negative (-) numbers.

In short, +> - which means positive is greater than negative. Even +1 is greater than -100.

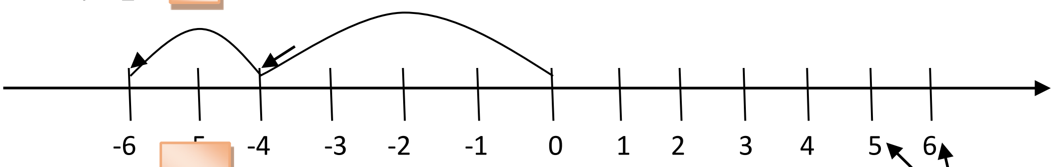
*To find answers using the number-line, you have to start from zero (0).



*When subtracting you move to the left side.

For example:

-4 – 2 =

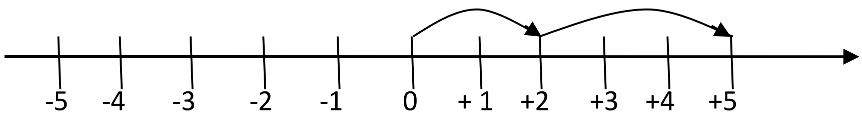


-4 – 2 = -6

Even without a positive sign, numbers on the right side of zero (0) are all positive numbers.

* -4 – 2 = -6 because you start from 0, then you move to four times to the left you reach -4, then again you move two times to the left and arrive at -6.

* When adding you move to the right. For example: 2+3 = 5



2 + 3 = 5, because when adding you move to the right side of the number-line. Start moving from 0 then move two steps and arrive at 2, then move three steps then you reach at 5, because the question says plus 3. The answer is 5 (positive 5).

*A number or letter which does not have a negative (-) or positive (+) sign, is a positive number or letter. In short, a number without any sign is a positive number.

For example:

- +5 is just the same as 5.
- +a is just the same as a.
- +24 is just the same as 24.

So whether you write +5 or 5 it is just the same thing. But when writing negative numbers, you should always write a negative (-) sign. If you forget to write a negative (-) sign on a negative number, then that number will not be a negative number, as a result, you will get a wrong answer.

For example, if you want to write negative 3 you have to write to -3.

*All addition and subtraction uses a number-line. However, there are questions that have big numbers. If you have very big numbers, you don't need to draw the number-line, you can use your mind online to imagine a number-line to find the answers.

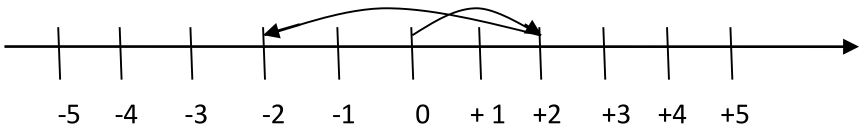
For example: -800 + 50 = -750

*When you are told to draw a number-line, you to show on a paper. However, especially in high school exams, you have to find your answers using your mind because you may waste time and space to draw the number-line in the exam. So, you should also learn to find answers quickly using you mind only. But 2 – 4 = -2

*Another simple example:

4 – 2 = 2

Here is how you find 2 – 4 = -2 using the number-line



1. -2 – (-3) =

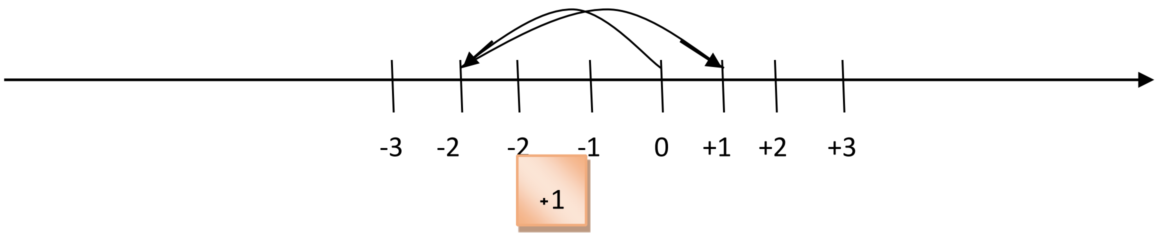
-2 – (-3) =

Brackets mean multiplication called X or times. So you can say:

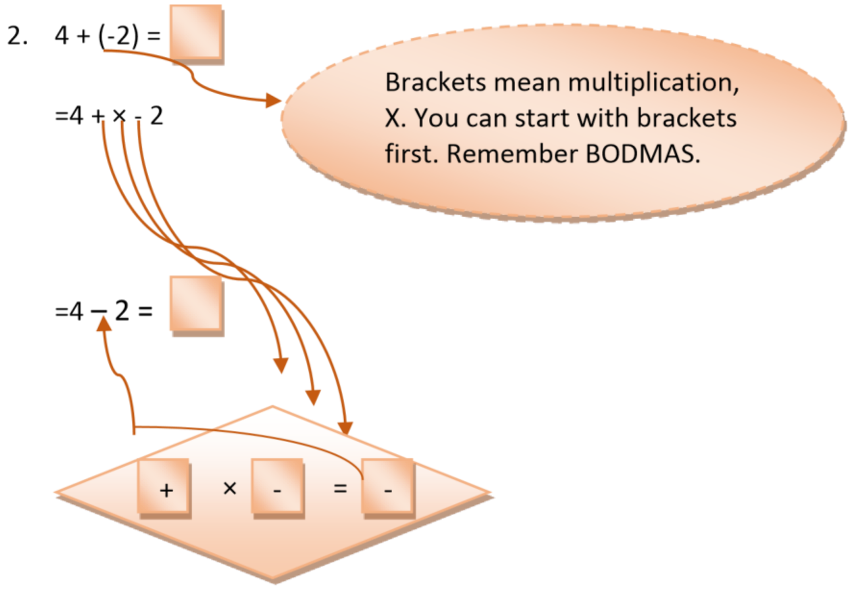
-2 – X – 3 or – X – =

So, -2 +3 = +1

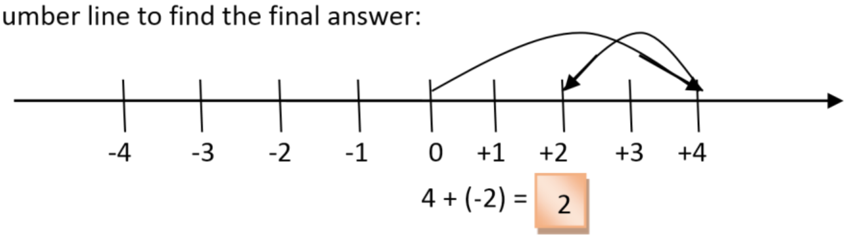
Then you can use the number line to show how to find the answer:



So, -2 – (-3) =



Then use a number line to find the final answer:



3. VARIABLES

- ❖ Variables are letters from a up to z. For example a, b, c, d, e, f, g, h, I, j, k, l, n, m, o, p, q, r, s, t, u, v, w, x, y, z.
- ❖ Many calculations in mathematics have letters (variables).
- ❖ You have to know how to calculate variables.
- ❖ In mathematics, questions that have letters come in four types (ways):
  1. Same variables like, x +x
  2. Variables with powers like, a^2 +a
  3. Different variables like, a-b
  4. Variables with numbers like, m × 2

Let's look at the four ways of calculating variables!

1. SAME VARIABLES □ ADDITION OF SAME VARIABLES

- ➢ You need to know that one variable or one letter has the value of 1. Meaning that a variable or a letter on its own has a value of 1. But, if it is with any number which is greater than 1, like 2b, it means b has the value of 2 not 1. 7x means that x has the value of 7.
- ➢ If two or more variables are added, their values are added together to find the answer. When writing the answer, only one variable is written with the answer. For example: a + a =2a

EXAMPLES

1. a + a +a = 3a

2. x + x + x = 3x

Because each letter has the value of 1. We have added the values of the letters together so, the answer is 3a. You do not need to write 3aaa, just write one letter a

SUBTRACTION OF SAME VARIABLES

- Firstly, you need to know how use a number line in order to subtract or add letters (variables).

- Subtraction depends on a number line, but sometimes it is not necessary to draw it all the time on a paper. Sometimes you only have to draw it in your mind.

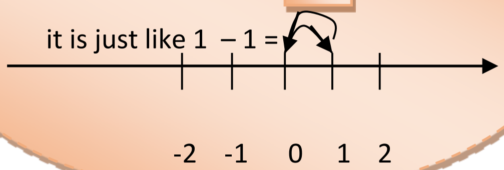
  For example: -1000 + 978 = -22

EXAMPLES

1. x + x = 0

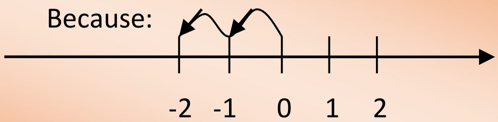
Because x has the value of one so, 0

it is just like 1 – 1 =



2. -x + x = -2x

Because:



So -x – x = -2x

- MULTIPLICATION OF SAME VARIABLES (×)
- When you multiply same variables, your answer will have a number at top right side.

  For example: x × x = x^2

  The number at the top right side of the answer is 2 because you have multiplied two X's.

  If there were three X's, the number on top would be 3.

  For example x × x × x = x^3

  Therefore, when multiplying same variables, the answer will have a number at the top according to the number of variables you will be multiplying.

1. $a \times a = \underline{a^2}$ 2. $a \times a \times a = \underline{a^3}$ 3. $b \times b = \underline{b^2}$

The small numbers at the top are called powers for example for a^2 it is power 2.

4. $x \times x \times x \times x = \underline{x^4}$
5. $y \times y = \underline{y^2}$
6. $m \times m \times m = \underline{m^3}$

SAME VARIABLES WITH POWERS

- Powers are numbers that you write at the top right side of a variable or at the top right side of any number. For example 10^2

X^3

These are powers

ADDITION OF SAME VARIBALES WITH POWERS

❖ Only same variables with same powers can be added together. For example:

1. $X^2 + x^2 = \underline{2x^2}$
2. $a^3 + a^3 = \underline{2a^3}$

Because we added two a^3

❖ In addition of same variables with powers, when writing an answer, write it with the same power given.
For example: $a^3 + a^3 = \underline{2a^3}$

❖ If the same variables have different powers, they cannot be added together.
For example: $a^2 + a^3 = \underline{a^2 + a^3}$

The answer is just the same as the expression given.

❖ If one variable has a power, whereas the other variable has no power, they cannot be added together

For example: $x + x^2 = \underline{x + x^2}$

The answer is just the same as the expression given.

2

The answer is just the same as the expression given because same variables with different powers cannot subtract

If a variable has np power, whereas the other one has a power, they cannot subtract.

2

The answer is just the same as the expression given because they cannot subtract

DIFFERENT VARIABLES

SUBTRACTION OF SAME VARIBALES WITH POWERS

- Just like in addition, only variables with same powers can subtract.
  For example: $x^2 – x^2 = \underline{0}$
- If same variables have different powers, they cannot subtract.
  For example: $a^3 – a^2 = \underline{a^3 – a}$

- For example: $b – b^2 = \underline{b – b}$
- Different variables cannot add.
  For example: $a + b = \underline{a + b}$
  The answer is just the same as the expression given.
- Different variables cannot add whether they have same powers or different powers.
  For example:
  1. $a^2 + b^2 = \underline{a^2 + b^2}$
  2. $a^3 + b^4 = \underline{a^3 + b^4}$

  The answers are just the same as the expressions given.
  Therefore, no addition of different variables! The same applies to subtraction.

- Different variables cannot subtract.
  For example: $a – b = \underline{a – b}$
  The answer is just the same as the expression given,
- Different variables cannot subtract whether they have same powers or different powers. For example:
  1. $a^2 – b^2 = \underline{a^2 – b^2}$
  2. $a^3 – b^4 = \underline{a^3 – b^4}$

  The answers found are just the same as the expressions given.
  Therefore, no subtraction of different variables.

MULTIPLICATION OF DIFFERENT VARIABLES

- Different variables can multiply whether they have same or different powers.
  For example:

1. $x \times x^6$ $= b^{2+3}$ $= \underline{x^3y^2}$

$= x^1 \times x^6$

$= x^{1+6}$

7

$= \underline{x}$

3. $x \times x \times x \times y \times y$

2. $b^2 \times b^3$

- If the variables are different, answers are written as follows:
  1. $x^3 \times a^2$ = $\underline{bm^3}$
     = $\underline{x^3a^2}$

3. $a^2 \times x$

  2. $b \times m^3$ = $\underline{a^2x}$

Remember BODMAS for expression

4 above.

- When finding answers while multiplying different variables, the variables are brought together. If the variables have powers, they are written with their powers.
  For example:
  $m^2 \times n$
  = m^2n

- IF THE VARIABLES ARE DIFFERENT, DO NOT ADD THE POWERS WHEN IN MULTIPLICATION! $a^2 \times b^3$
  = $\underline{a^2b^3}$

- When writing the answer, each variable is written with its power.
  But if the variables are the same you can add the powers.
  For example:
  $x^2 \times x^3 = x^{2+3} = \underline{x^5}$

VARIABLES AND NUMBERS

1.ADDITTION OF SAME VARIABLES WITH NUMBERS

EXAMPLES:

1. $2b + 3b = \underline{5b}$ 3. $17x^3 + 10x^3 = \underline{27x^3}$
2. $6a^2 + 3a^2 = \underline{9a^2}$ 4. $3m^2 + 3b^2 = \underline{6m^2}$

- Same variables with numbers having SAME POWERS can add.

4. $m + m \times a^2$
   = $\underline{m + ma^2}$

- Same variables with numbers BUT having different powers CANNOT add.
  For example:
  $5x^2 + 5x^3 = 5x^2 + 5x^3$
  The answer is the same as the expression given.
- A variable with a number and a number without a variable cannot add.
  For example: $3x + 3 = \underline{3x + 3}$
  The answer will be same as the expression given.

DIFFERENT VARIABLES WITH NUMBERS CANNOT ADD EXAMPLES

1. $2b + 2x = \underline{2b + 2x}$
2. $3x + 4y = \underline{3x + 4y}$
   The answers found are just the same as the expressions given 1 in itself.

<u>SUBTRACTION OF SAME VARIABLES WITH NUMBERS HAVING POWERS</u>

EXAMPLES

1. $9x^2 - 6x^2 = \underline{3x^2}$ 3. $4q^4 - 3q^4 = \underline{q^4}$ 5. $3a^2 - a^2 = \underline{2a^2}$
2. $3x^3 - 2x^3 = \underline{x^3}$ 4. $2a^3 - a^3 = \underline{a^3}$ 6. $9x - 9x = \underline{0}$

- Just like in addition, same variables with numbers having same powers can subtract.
- In subtraction, when writing an answer, you write it with the same power given in the expression.

For example: $6x^2 - 2x^2 = \underline{2x^2}$

- Same variables wit numbers BUT HAVING DIFFERENT POWERS CANNOT SUBTRACT.

For example: $2x^3 - 2x^4 = \underline{2x^3 - 2x^4}$

The answer is just the same as the expression given.

- ❖ A variable with a number cannot subtract or cannot be solved with a number which does not have a variable.

For example: $3a - 3 = \underline{3a - 3}$

- ❖ But a variable with a number can subtract with a variable without a number, if the variables are the same.

For example: $3a - a = \underline{2a}$

- ❖ DIFFERENT VARIABLES WITH NUMBERS CANNOT SUBTRACT

For example $3x - 2y = \underline{3x - 2y}$

5.EQUALITIES AND INEQUALITIES

- ❖ This is one of the simple topics that everyone should understand in order to know mathematics.

A.EQUALITIES

- ❖ The name equality comes from the word equal, which is written as =.
- ❖ This topic involves expressions that have an equal sign.

EXAMPLES

1. $2 + 3 = x$

   Find the value for x, or find the number for x. 4.

   $2 + 3 = x$

   $5 = x$ or you can write it as $x = 5$

2. $x = 5 - 2$

$\rightarrow x = 3$

3. $2 + x = 4$

- ❖ Here we have to put some things in order or together.
- ❖ When solving, always put numbers on the same side and variables (letters) on the other side of the equal sign.
- ❖ When you take a number or variable to the other side of the equal (=) sign, its sign or operation changes. If it was negative (-) it becomes positive (+). Of it was positive (+) it becomes negative (-).

- ❖ *Let's solve number 3!*

5.

$2 + x = 4$ *2 will jump to the other side of the equal sign, so it will become -2, but,

*A negative sign will be written as minus (–), because we are still solving.

*But when writing the answer, just write a small negative (e.g. -2)

*if it is a positive number write 2 or +2.

$y + y = 4$

$\rightarrow 2y = 4$

Since there is nothing between 2 and y, 2 cannot cross over the equal sign, so we use division or 'over.' 2 over 2 on both side to leave y alone. This is what you should do to all such expressions. When you want to remain with a variable alone when it has a number behind it without any operation between the number and the variable, you should not take the number to the other side of an equal sign, you just have to divide with same number on both sides of an equal sign.

$\frac{\not{2}y}{\not{2}} = 4$

$\rightarrow y = 4$

$3x + 4 = x - 2$

$\rightarrow 3x + 4 = x - 2$

$\rightarrow 3x - x = -4 - 2$

$\rightarrow 2x = -6$ using

$\rightarrow x = 3$

Because x was positive, but it jumped over an equal (=) sign
so, it becomes negative,
which is minus (–) because

6. -2x = -4

$\rightarrow \frac{-2x}{-2} = \frac{-4}{-2}$

$\rightarrow x = 2$

7. -4y = 4

$\rightarrow \frac{-4y}{-4} = \frac{4}{-4}$

$\rightarrow y = -1$

1. 3 – x = 4 + x

$\rightarrow -x - x = 4 - 2$

$\rightarrow -2x = 2$

$\rightarrow \frac{-2x}{-2} = \frac{2}{-2}$

$\rightarrow$ <u>x = -1</u>

<u>B.INEQUALITIES</u>

- Inequalities involve these operations ≤ and ≥

EXAMPLES

1. $-2 + y \leq 4 \rightarrow$

$-2 + y = 4$

$\rightarrow y = 4 + 2$

$\rightarrow y = 6$

- ❖ When negatives cancel each other, the operations $\geq$ or $\leq$ change direction.

  For example:

  $4 - x \geq x - 2$

  $\rightarrow 4 - x \geq x - 2$

  $\rightarrow -x - x \geq -4 - 2$

  $\rightarrow -2x \geq -6$ using the number line.

  $\rightarrow \frac{-2x}{-2} \geq \frac{-6}{-2}$

  $\rightarrow x \leq 3$

  This sign has changed direction, it was $\geq$, but now it has become $\leq$ because the negatives have cancelled.
- ❖ Only change the direction of the sign $\geq$ or $\leq$ if you have cancelled the negatives on both sides.
- ❖ If you have only cancelled the negatives on one side, you cannot change the direction if the sign.

RESULTS

After implementing the guidelines in this study on students who had struggles understanding mathematics, there was a significant improvement in their performance. These guidelines are useful as they can enable students to easily solve mathematics from any book just by applying the rules and following the examples given, even without the help of a teacher.

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BBC.uk(2015) Order of operations. Retrieved: July 27, 2015, from the BBC Website: http://www.bbc.co.uk/bitesize/standard/maths_i/numbers/order_of_op/revision/1/