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Using Vector and Conventional Approach in Calculating Distance in a Three Dimensional Objects: An Experimental Study

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ABSTRACT Many of high school students find difficulty in studying three dimensional geometry. They find difficulty in understanding three dimension objects such as cube, cuboid, pyramid, etc. Most students fail to make correct perception about the distance to calculate, when the problems are dealing with lines or planes. Only students with good spatial capability can solve such problems. This research focused on making new approaching method to understand and then calculate the distance in a three dimensional objects. The approach is vector approach, including develop connection vectors, normal vector of a plane, and calculating distance from a point to certain line or plane using vector theorems. The Approach was tested by experimental study to a class of XII grade of high school students in SMAN 1 Parongpong, Indonesia, consists of 30 students, and conducted in six meeting during July until August 2017. A similar class treated with the conventional spatial approach as the control class. SPSS 20 was used to analyze the research data. Based on the research data analysis, the conclusion of the study was that the students treated with vector approach achieve higher performance improvement compare with the students treated with conventional approach. By using vector theorems to solve three dimensional problems, we turn the theory into action.

Keywords : *Vector Approach, Three Dimensional Objects, Distance, Line, Plane*.

1. Introduction

1.1. Background Of The Study

In the High School level Mathematics curriculum in Indonesia, the Three Dimension is one of the subjects that must be taught to students. The problems related to the subject also always appear both in the National Examination and in the entrance test for Higher Education. There, the students learn how to calculate the distance and angle between the elements of geometry contained in various types of three dimensional objects, including cubes, rectangular prisms, and pyramid. The geometrical elements in such problems are points, lines and planes.

To solve the distance and angle problems, all of the high school mathematics textbooks in Indonesia use the spatial approach, and the calculation use conventional formulas, including Pythagoras formula, triangle area formula, trigonometric formulas, and geometric congruency formula. The approach can easily found in Sukino (2012) and Kemendikbud Indonesia (*Matematika Kelas XII Kurikulum 2013 Revisi 2018*, n.d.) The spatial approach consist of several steps. It begin with simplifying the plane into a line, then simplify the problem of three dimensions (space) into a two-dimensional problem (plane), and end with the use of conventional formulas above. Until now, there is no book that discusses the use of vector and matrix approaches to analyze and solve the problem of calculating distance and angle.

With the spatial approaches and conventional formulas, researchers found many complaints from students regarding the problem of calculating these distances and angles. Some complaints that often appear include:

1. Students are not sure of their own geometrical interpretation when simplifying three-dimensional problems into two-dimensional problems.
2. It is quite difficult to imagine the position of the line or plane intended by practice questions or exam questions, especially in imagining the location of the line of projection of a line on a particular plane.

3. The time to work on a three-dimensional question is more than 5 minutes, while the average time to do the National Examination questions should be 3 minutes.

These difficulties and complaints led to a decline in students' interest in the Three Dimensions chapter in particular, and geometry in general.

Thus, a new approach is needed to be taught to students who have difficulty using the spatial approach when calculating distance in a three dimensional solid.

1.2. Research Question

Based on the background of the study, the questions to be answered through this research are

(1) Is there any significant improvement in problem solving ability in students who are given a vector approach?
(2) Are there any significant differences in problem solving ability improvement of students who are given a vector approach to students who are given a spatial/conventional approach?

2. Methodology

This chapter gives an overview of research variables, research design, and research instruments.

2.1. Research Variables

The variables in this study are

(1) Dependent Variable : problem solving abilities of the students in three dimensions geometry, especially the distance calculating problems.
(2) Independent Variables : the use of vector approach and (vs.) conventional spatial approach.

2.2. Research Design

This research is an experimental quantitative study of 12th grade of high school students in Parongpong Public High School, in particular Science Class 1 as the experiment group (EG) and Science Class 2 as the control group (CG). Each class consists of 30 students, and the initial abilities of the two classes are not significantly different. The Figure 2.1 show the flowchart of the study.

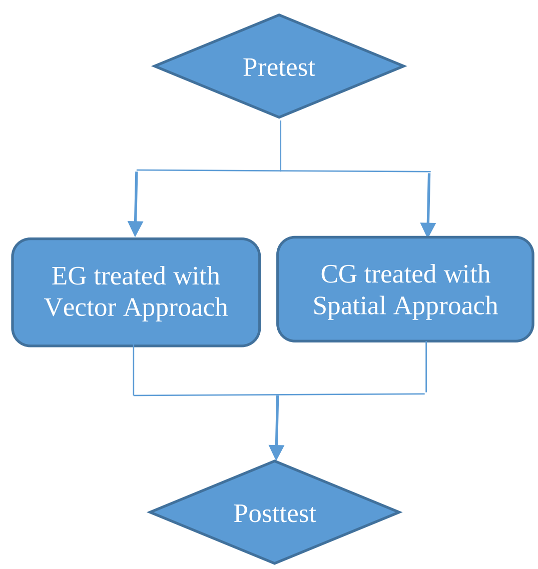

Figure 2.1 *Flowchart Of The Study*

2.3. Research Instrument

The research was conducted in six weeks, from July 2018 until August 2018 , at Public High School I Parongpong, West Bandung, West Java Province.

Starting with the pretest to students both in the experimental class and in the control class. Pretest consists of five questions, including the problem of distance between points, distance from point to line, and distance from point to flat plane.

In subsequent meetings the experimental class receives the three-dimensional learning using a vector approach, while the control class uses a conventional spatial approach. Especially in the experimental class matrix and vector theorems are also taught, because there are many students who forget about the theorems that have been taught in the 10th grade.

The study ended with a post test that was identical to the pre test. Both pre-test and post-test have an essay form, and are assessed based on the assessment rubric created by the researcher.

The test results of the control group and experimental group were processed using SPSS 20 to obtain normalized gain for each group. The gain will be differentiated according to its categories based on Hake (2002), i.e.

(1) low category if the gain is less than 30%,
(2) medium category if the gain is between 30% to 70%,
(3) high category if the gain is more than 70%.

3. Vector Approach vs Spatial Approach

This chapter gives brief overview on the aproaches used in the research.

According to Rahadi (2018), the vector approach to calculate distance in a three dimensional objects consists of

1. Formation of vectors in the three dimension objects.
2. Formation of normal vectors in the planes located inside cubes or prisms.
3. Operation of adding and subtracting vectors
4. Dot products
5. Cross product
6. Orthogonal projection of a vector to another vector
7. Phytagoras.

The problem of distance in space in three dimensions can be classified as follows

(1) Distance between two points
(2) Distance from point to line
(3) Distance from point to flat plane
(4) The distance between two parallel lines
(5) The distance between two parallel planes
(6) Distance of lines to parallel planes
(7) The distance between two lines that are not intersecting but not parallel.

As explained in Indonesia (2018), the spatial approach to calculate distance in three dimensional objects consist of Phytagoras formula, geometry congruency principle, triangle area formula, and simplifying the problems into two dimensional objects. The same steps can also be found in Sukino (2012).

In order to make clear the difference and similarity, the two approaches in this study will be used to solve the problems in example 3.1 and 3.2.

Example 3.1. Given the cube ABCD.EFGH with the edge length of $6\,cm$. The point P is the midpoint of edge DH as ilustrated by Figure 3.1. Calculate the distance from point P to the plane ACH.

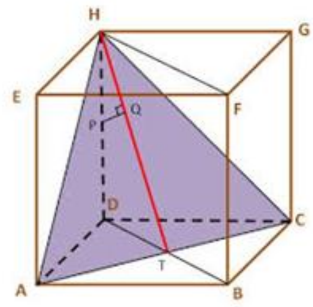

Figure 3.1 *Cube ABCD.EFGH*

Solution by spatial approach

Because the ACH triangle is an equilateral triangle, it can be represented by the median line, the HT line, with the point T being the midpoint of the AC. Thus, the distance from P to the line HT represent the distance from point P to the plane ACH.
Consider right triangle HDT in the Figure 3.2, with point Q is the projection of point P onto HT, and R is the projection of point D onto HT.
Easy to find that $DT = 3\sqrt{2}$, and by Phytagoras theorem $HT = 3\sqrt{6}$.
Triangle area equality principle used to find the length of DR.

$$[HDT] = [HDT]$$
$$\frac{HT \cdot DR}{2} = \frac{DT \cdot DH}{2}$$

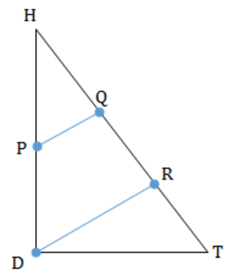

Figure 3.2 *Right triangle HDT*

$$3\sqrt{6} \cdot DR = 3\sqrt{2} \cdot 6$$
$$DR = 2\sqrt{3}$$

It is obvious that the DRH triangle is similar with the PQH triangle, so based on the similarity theorem, PQ length is half of the length of DR.

$$PQ = \frac{1}{2} DR = \sqrt{3}$$

So, the distance of point P to the plane ACH is $\sqrt{3}\ cm$.

Solution by vector approach

The direction rule in this solution following the Rahadi rule. The vector HP is chosen as the connector of point P to plane ACH.

$$\overrightarrow{HP} = \begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix}$$

Easy to find that $\overrightarrow{CF}$ is normal to plane ACH.

$$\vec{N} = \overrightarrow{CF} = \begin{pmatrix} 6 \\ -6 \\ 6 \end{pmatrix}$$

The distance of point P to the plane ACH is calculated as the scalar projection of the connection vector to the normal vector

$$\overrightarrow{HP}_N = \left| \frac{\overrightarrow{HP} \cdot \vec{N}}{|\vec{N}|} \right| = \left| \frac{\begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -6 \\ 6 \end{pmatrix}}{6\sqrt{3}} \right|$$

$$\overrightarrow{HP}_N = \frac{18}{6\sqrt{3}} = \sqrt{3}$$

So, the distance of point P to the plane ACH is $\sqrt{3}\ cm$.
Both approach give the same result.

Example 3.2.
Given a square pyramid T.ABCD with base length of b cm, and the lateral edge length of l cm as shown in Figure 3.3. Point M is the midpoint of TB. Determine the distance of point D to point M.

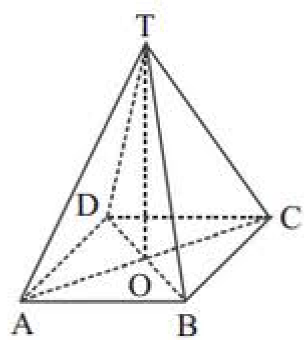

Figure 3.3 Right square pyramid T.ABCD

<u>Solution by spatial approach</u>

The solution started with finding the pyramid height using phytagoras formula on the right triangle TOD, considering diagonal $DB = b\sqrt{2}$, and DO is half-length of DB.

$$h = \sqrt{DT^2 - DO^2}$$

$$h = \sqrt{l^2 - \left(\frac{b\sqrt{2}}{2}\right)^2}$$

$$h = \sqrt{l^2 - \frac{1}{2}b^2}$$

Consider isosceles triangle TDB to find namely point N, the projection of the point M to the line DB, as shown in the Figure 3.4.

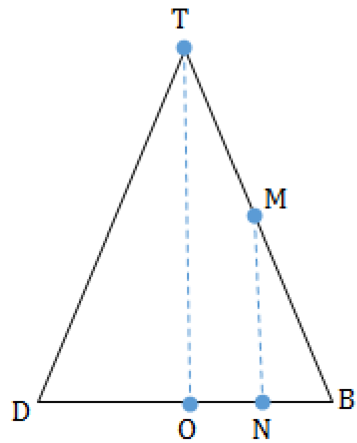

Figure 3.4 *Isosceles triangle TDB*

Because M is the midpoint of TB and by similarity principle, MN is half-length of TO

$$MN = \frac{1}{2}TO = \frac{1}{2}h.$$

Also, by similarity principle, BN is half-length of BO, implies that DN is third-quarter of DB

$$DN = \frac{3}{4}DB = \frac{3}{4}b\sqrt{2}.$$

The distance of point D to point M is simply calculated by phytagoras formula of right triangle DNT

$$DM = \sqrt{DN^2 + MN^2}$$

$$= \sqrt{\left(\frac{3}{4}b\sqrt{2}\right)^2 + \left(\frac{1}{2}h\right)^2}$$

$$= \sqrt{\frac{9}{8}b^2 + \frac{1}{4}l^2 - \frac{1}{8}b^2}$$

$$DM = \sqrt{b^2 + \frac{1}{4}l^2}.$$

<u>Solution by vector approach</u>

The pyramid height will be calculated using the magnitude formula of the vector DT.

$$\overrightarrow{DT} = \begin{pmatrix} b/_2 \\ -b/_2 \\ h \end{pmatrix}$$

$$l = |\overrightarrow{DT}| = \sqrt{\frac{b^2}{2} + h^2}$$

$$h = \sqrt{l^2 - \frac{1}{2}b^2}.$$

Exactly the same result with the conventional approach.
Because M is the midpoint of TB, by colinear principle

$$\overrightarrow{DM} = \frac{1}{2}\overrightarrow{DT} + \frac{1}{2}\overrightarrow{DB}$$

$$= \frac{1}{2}\begin{pmatrix} b/_2 \\ -b/_2 \\ h \end{pmatrix} + \frac{1}{2}\begin{pmatrix} b \\ -b \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 3b/_4 \\ -3b/_4 \\ h/_2 \end{pmatrix}$$

The distance of D to M is

$$|\overrightarrow{DM}| = \sqrt{\frac{9}{8}b^2 + \frac{1}{4}h^2}$$

$$|\overrightarrow{DM}| = \sqrt{b^2 + \frac{1}{4}l^2}.$$

Again, both approaches give exactly the same result.

Table 4.1 *Descriptive Statistics for The Experiment Group (EG) and The Control Group (CG)*

	Vector Approach (EG)		Spatial Approach (CG)	
	Mean	S.D.	Mean	S.D.
Pretest	10.8000	3.54673	11.1333	2.90936
Posttest	47.9667	19.37735	16.7000	15.54781
Normalized Gain	0.4100	0.22151	0.0460	0.17700

4. Results and Findings

To answer the first question, pretest and posttest are conducted, and the scores was calculated using IBM SPSS 20. The results are shown in the Table 4.1.

As shown in Table 4.1 the average score of students in the control class was 11.1 when the pre-test, and slightly increased to 16.7 during the post test. The increase provided a normalized gain of 4%, which is categorized as low gain according to Hake gain categorization.

Based on the test answer sheets, more than 75% of the control class students make geometric interpretation errors on the geometry elements needed to solve the problems given in the test. Researchers also found that there were some students who still did not understand what to do to solve the three dimensional problems given in the test.

Meanwhile, the students in the experimental group who are given vector approach achieve higher improvement in their ability to solve three dimensional distance problems. Their average score increase from 10.80 in the pre-test to 47.97 in the post-test. It also shown that the experimental group reaches gain 41%, which is categorized as medium category based on Hake.

Since the normalized gain was normally distributed and the data is homogenous, then the independent sample t-test was done to answer the second question in 1.2. The result of the t-test shown in the Table 4.2.

As shown in the Table 4.2, the significance of the test is 0.000 , less than $\alpha = 0.005$. It means the null hypotesis is rejected, in another word the two groups have significantly different gain, with 95% confidence interval of the difference.

Furthermore, considering the experimental group has higher gain than the control group based on the Table 4.1, it can be concluded that the vector approach producing higher problem solving ability improvement in three dimensional problems rather than the conventional/spatial approach among the high school students.

Table 4.2. *T-Test for Equality of Means between the EG and the CG*

Levene's Test for Equality of Variances		t-test for Equality of Means ($H_0 : \mu_1 = \mu_2$)						
F	Sig.	T	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
							Lower	Upper
1.955	.167	7.030	58	.000	.36392	.05177	.26030	.46754

5. Conclusions and Recommendation

Based on the results and the data analysis, the conclusions of the research are

(1) The students who were given a vector approach experienced an improvement in Dimension Three problem solving abilities, in the medium improvement category.

(2) The students who were given a vector approach reach significant higher improvement in Dimension Three problem solving abilities rather than the students who were given a conventional spatial approach.

In order to improve the quality of the approach, the research recommendations are

(1) It is necessary to establish matrix and vectors theorems before the vector approach is taught to students, because several students already forgot what they have learned about matrix and vector in the 10^{th} grade.

(2) Interactive learning tools and geometry learning media are needed to be created to make it easier for students to master this vector approach.

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